References

[1] E. Santopinto, J. Ferretti, Phys. Rev. C, 92(2015)025202.

[2] N. Isgur, Phys. Rev. D, 62(2000)014025.

[3] K. A. Olive, (Particle Data Group), Chin. Phys. C, 38(9)(2014)090001.

1 - 17 New Topological Structure in Skyrme Model*

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Recently it has been pointed out that the skyrmions carry two independent topologies, the baryon topology and the monopole topology[1]. According to this philosophy, the baryon number B of skyrmions could be decomposed to the monopole number m and shell (radial) number n, so that the baryon number is given by $B=mn$. This indicates that the skyrmion should be described by two types of topology. In this letter we show that baby skyrmions can also be generalized to have two topologies $\pi_1(S^1)$ and $\pi_2 S^2$, and thus they are classified by two topological numbers m and n. This confirms the result in Ref. [1] that skyrmions carry two independent topologies.

The Skyrme theory has been proposed as a theory of pion physics in strong interaction where the skyrmion, a topological soliton made of pions, appears as the baryon[2]. The Lagrangian of Skyrme theory could be written as

$$\begin{aligned}\mathcal{L}&=\frac{\kappa^2}{4}\mathrm{tr}\ L_\mu^2+\frac{\alpha}{32}\mathrm{tr}([L_\mu,L_\nu])^2\\&=-\frac{\kappa^2}{4}\left[\frac{1}{2}(\partial_\mu\omega)^2+2\sin^2\frac{\omega}{2}(\partial_\mu\hat{n})^2\right]-\frac{\alpha}{8}\left[\sin^2\frac{\omega}{2}\left((\partial_\mu\omega)^2(\partial_\nu\hat{n})^2-(\partial_\mu\omega\partial_\nu\omega)(\partial_\mu\hat{n}\cdot\partial_\nu\hat{n})\right)+2\sin^4\frac{\omega}{2}(\partial_\mu\hat{n}\times\partial_\nu\hat{n})^2\right],\end{aligned}\tag{1}$$

where $L_\mu=U\partial_\mu U^\dagger, U=\exp(\frac{\omega}{2i}\vec{\sigma}\cdot\hat{n})=\cos\frac{\omega}{2}-i(\vec{\sigma}\cdot\hat{n})\sin\frac{\omega}{2}$ and w, $\hat{n}$ are the massless scalar field and pion field respectively.

With an interesting limit

$$w=(2n+1)\pi.\tag{2}$$

The Skyrme theory is reduced to Skyrme-Faddeev theory

$$\mathcal{L}=-\frac{\kappa^2}{2}(\partial_\mu\hat{n})^2-\frac{\alpha}{4}(\partial_\mu\hat{n}\times\partial_\nu\hat{n}),\tag{3}$$

where $\hat{n}$ could be viewed as the CP^1 filed which carries the topology $\pi_2(\mathrm{S}^2)$. The soliton solutions in this theory with topology $\pi_2(\mathrm{S}^2)$ is called as baby skyrmion. The baby skyrmion could be obtained by the ansatz[3]

$$\hat{n}=\begin{pmatrix}\sin f(\varrho)\cos m\varphi\\ \sin f(\varrho)\sin m\varphi\\ \cos f(\varrho)\end{pmatrix}.\tag{4}$$

which has the equation of motion

$$\left(1+\frac{\alpha}{\kappa^2}\frac{m^2}{\varrho^2}\sin^2 f\right)\ddot{f}+\frac{1}{\varrho}\left(1+\frac{\alpha}{\kappa^2}\frac{m^2}{\varrho}\dot{f}\sin f\cos f-\frac{\alpha}{\kappa^2}\frac{m^2}{\varrho^2}\sin^2 f\right)\dot{f}-\frac{m^2}{\varrho^2}\sin f\cos f=0.\tag{5}$$

One could obtain the baby skyrmion solution by solving above equation with boundary condition $f(0)=\pi, f(\infty)=0$.

However, one should notice that the baby skyrmions are described by the filed $\hat{n}$, and without the filed w. As a result, the baby skyrmions carry only one topology number that determined by $\hat{n}$. However, as we claimed at the beginning, the baby skyrmion could also be described by two types of topology. In this way, one should activate the role of w in the model, and generate the ansatz to

$$w=w(\varrho),\quad \hat{n}=\begin{pmatrix}\sin f(\varrho)\cos m\varphi\\ \sin f(\varrho)\sin m\varphi\\ \cos f(\varrho)\end{pmatrix}.\tag{6}$$