# 1 - 18 Photon Polarization Tensor in a Magnetized Plasma System*

Chao Jingyi and Huang Mei

The photon polarization tensor carries the fundamental information of magnetized vacuum or medium[1−5]. A complete description of the vacuum polarization tensor is particularly complicated to approach, since the vacuum photon polarization tensor is expressed in terms of a double summation of infinite series with respect to two Landau levels occupied by virtual charged particles. Most works were focusing on the strong filed limit with an assumption of Lowest Landau Level (LLL)[2, 6]. In Ref. [7] we obtained a full description of vacuum polarization tensor in response to all the Landau levels at any field strength of $B$ for the first time beyond LLL approximation, and we found out that the imaginary part of the photon polarization tensor $\Im\Pi$ becomes nonzero at the time like momenta region $Q^2>4(M^2+2neB)$ at $T=0$, *i.e.*, the LLL approximation is analytically satisfied[3, 7].

It was not fully understood of the imaginary parts of thermal photon polarization tensor in a magnetized media. The main purpose of this paper is to investigate whether the above conclusion will be influenced by temperatures, which, in turn, is not keeping suitable. At finite temperature, in our scheme, the Matsubara frequency was summed after applying Poisson summation formula, which finished easily and convergent quickly under the help of proper time representation. It was argued that the summation of Matsubara frequency is not commuted with Landau level ones[8]. Our calculation exclude this conjecture. The summation of Landau level is convergent as it has to be since no more divergent environment is included. However, in the early works, one has to test by a numerical way to find out the cutoff of the Landau level. In our work, the dependence of Landau level is expressed in an obviously matter. Therefore, one is able to truncate the Landau level via a systematic consideration while proceeding numerical simulation.

The photon polarization tensor is expressed as $\Pi^{\mu\nu}(q)=-\mathrm{i}e^2\mathrm{Tr}[\$(k)\gamma^\mu\$(p)\gamma^\nu]=\sum_{i=1}^4 P_i^{\mu\nu}\pi_i$. We found two branch cuts along the real axis of $q_0$. After extended it to the complex plane, one has $\mathrm{Disc}\,\pi_i(q_0)=\pi_i(q_0+\mathrm{i}\varepsilon)-\pi_i(q_0-\mathrm{i}\varepsilon)=2\mathrm{i}\,\Im\pi_i(q_0)$. In the strong magnetic field and chiral limit, we write down the discontinuities directly.

The first branch cut, $q_0^2<q_3^2+2M_n^2-2\sqrt{M_n^4+q_3^2M_n^2}<q_3^2$, is developed at finite temperatures, which is corresponding to the process of $\gamma+\psi\leftrightharpoons\psi$. One has:

$$\begin{aligned}
&\mathrm{Disc}\,\pi_1(q_0)\simeq 2\,\Im\left\langle\left(M^2+\eta q_{\shortparallel}^2\right)\left(\Xi_0^T(v)+\Xi_0^T(v+1)\right)+(2eB)\left(\Xi_2^T(v)+\Xi_2^T(v+1)+\Xi_4^T(v)+\Xi_4^T(v+1)+2\Xi_0^T(v)\right)\right\rangle\\
&\simeq\left(\sum_{n=0}^{j}+\sum_{n=1}^{j}\right)\frac{-1}{4\pi^{\frac{3}{2}}}\frac{(2eB)\left(q_{\shortparallel}^4-4q_0^2M_n^2\right)^{\frac{1}{2}}}{T^{\frac{1}{2}}|q_0|^{\frac{3}{2}}}\mathrm{Li}_{-1}\left(-\mathrm{e}^{-\frac{q_3^2}{2|q_0|T}}\right)\\
&\mathrm{Disc}\,\pi_2(q_0)\simeq\frac{q_\perp^2}{2eB}\mathrm{Disc}\,\pi_1(q_0);\qquad \mathrm{Disc}\,\pi_3(q_0)\simeq-\frac{q_\perp^2}{3q^2}\mathrm{Disc}\,\pi_1(q_0);\qquad \mathrm{Disc}\,\pi_4(q_0)=0.
\end{aligned}\tag{1}$$

Where $\mathrm{Disc}\,\Pi^{\mu\nu}(q_0)=\sum_{i=1}^4 P_i^{\mu\nu}\,\mathrm{Disc}\,\pi_i(q_0)$ as denoted before. Roughly speaking, the finite-$n$ Landau level contribution is supposed to exponentially suppressed, $\mathrm{e}^{-n}$, which is the underlying of the approximation of LLL. But, in a strictly manner, the $n$-th Landau levels present as $L_n^{(\alpha)}\mathrm{e}^{-n}$. Indeed, the Laguerre polynomial was neglected improperly in lots of early works. For large $n$, the asymptotic behavior of $L_n^{(\alpha)}$ is limit to $n^{\frac{\alpha}{2}-\frac{1}{4}}$. Hence, $L_n^{(\alpha)}\mathrm{e}^{-n}$ is characterized by a non-monotonic behavior of $n$ when $\alpha\geq 1$. In other words, the LLL approximation will break down if the terms contained $L_n^{(1)}$ play an important role in the estimation.

The second branch cut, $q_0^2>q_3^2+4M^2$, is due to the conventional process $\gamma\leftrightharpoons\psi+\psi$. In the chiral limit $M\to 0$, one has:

$$\begin{aligned}
&\mathrm{Disc}\,\pi_1(q_0)\simeq 2\,\Im\left\langle\left(\eta q_{\shortparallel}^2\right)\Xi_0^T(v)-(2eB)\left(\Xi_2^T(v)+\Xi_4^T(v)\right)\right\rangle=\frac{(2eB)|q_{\shortparallel}|^3}{2^{11}\cdot\pi T^3};\\
&\mathrm{Disc}\,\pi_2(q_0)=0;\qquad \mathrm{Disc}\,\pi_3(q_0)=-\frac{q_\perp^2}{q^2}\mathrm{Disc}\,\pi_1(q_0);\qquad \mathrm{Disc}\,\pi_4(q_0)=0.
\end{aligned}\tag{2}$$

It is well known that the self-energies of gauge bosons take the same forms for both QED and QCD plasmas in the limit of long wavelength. It takes place in the strong magnetic filed limit, as well. In the space like momenta regime $q_3^2\gg q_0^2$ and strong $B$-fields limit $(2eB)\gg q_0^2,q_3^2$, it is allowed us to define $(2eB)\sim\lambda T^2$ and $\lambda>1$. The classification of the energy scale is similar to hard-loop action, where loop momenta $k\sim M_n$ for finite-$n$ Landau levels while as the external momenta $q_3\sim\lambda^{-\frac{1}{2}}T^{\frac{1}{2}}M_n^{\frac{1}{2}}$, $q_0\sim\lambda^{-\frac{3}{2}}T$. Described by the result of Eq. (1), $\mathrm{Disc}\,\pi_1$ is at

* Foundation item: MOST(2015CB856903)

the order of $\lambda^{\frac{7}{4}}\mathrm{Li}_{-1}(-\mathrm{e}^{-\lambda})$, which is not monotonically decreasing as $\lambda$ increasing. Physically, such unique feature are essentially same as other gauge theories governed plasma systems. This large imaginary part only arises at finite temperatures, which is the so called Landau damping. It demonstrates the absorption of soft fields by hard plasma constituents. Such kind of processes gains the contribution not only from the lowest (soft) Landau level but also up to the finite-$n$ (hard) levels. The general mathematic explanations have been analyzed as before. Eventually, we conclude that the LLL approximation is suit at zero temperature but not well described at finite temperatures. In particular, the hard-loop approximation takes control for the magnetized plasma systems, whose constituent of typical momentum is much larger than the probing wave vector.

## References


[1] D. B. Melrose, R. J. Stoneham, Journal of Physics A: Mathematical and General, 10(7)(1977)1211.

[2] G. Calucci, R. Ragazzon, Journal of Physics A: Mathematical and General, 27(6)(1994)2161,

[3] Koichi Hattori, Kazunori Itakura, Annals of Physics, 330(2013)23.

[4] Felix Karbstein, Phys. Rev. D, 88(8)(2013)085033.

[5] Ken-Ichi Ishikawa, Daiji Kimura, Kenta Shigaki, et al., International Journal of Modern Physics A, 28(2013)50100.

[6] V. P. Gusynin, V. A. Miransky, I. A. Shovkovy, Nucl. Phys. B, 462(2)(1996)249.

[7] J. Chao, L. Yu, M. Huang, Phys. Rev. D, 90, 4(2014)045033. Erratum: [Phys. Rev. D, 91(2015)029903.] doi:10.1103/Phys. Rev. D, 90.045033, 10.1103/PhysRevD.91.029903 [arXiv:1403.0442 [hep-th]].

[8] E. Elizalde, Emili, Zeta regularization techniques with applications. World Scientific, (1994).