2 - 23 Fractal Geometrical Properties of Nuclei

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The nuclear system may be a fractal object with the characteristic of self-similarity, the nuclear irregular structure properties and the self-similarity characteristic are considered to be an intrinsic aspects of nuclear structure properties. For the description of nuclear geometric properties, nuclear fractal dimension is an irreplaceable variable similar to the nuclear radius.

In our present description, a more general conception of nuclear fractal clusters (NFCs) is applied, which is similar but different from the conventional one of the α-cluster structure and the core plus valence nucleons structure in halo nuclei. The latter is considered to be one kind of the former which is based on the concept of the characteristics of fractal objects. The concept of NFCs is that of the nucleus as a fractal assembly of structural subunits that are themselves made up of no less than one nucleon and keep certain correlation of the similarity with the ensemble in the geometrical and in physics. And the geometrical boundary of the NFCs within some nuclei is less distinguishable than that of the clusters in α-cluster nuclei and the halo nuclei.

Similar to the definition given in[1], an isotropic self-similarity nuclear fractal dimension D_f is defined in the following relation $M(b{\cdot}r)=b^{D_\mathrm{f}}{\cdot}M(r)$, where $M(r)$ is the mass number within the size r of the fractal object; $M(b{\cdot}r)$ is the mass number of b times the size r of the fractal object, where b is a scaling factor among the similar parts within the fractal object. The only solution for relation (1) is $M(r)\propto r^{D_\mathrm{f}}$. The nuclear average matter density $\rho(r)$ with the law of decay of isotropic spatial correlation, $\rho(r)=\frac{M(r)}{V(r)}\propto\frac{r^{D_\mathrm{f}}}{r^3}\propto r^{D_\mathrm{f}-3}$, is a basic variable function in nucleus. So far, the geometric dimension of nuclei is considered as 3, because of the concept of liquid drop model. For a real physical nuclear object embedded in 3-dimension Euclidean-space, its dimension must be less than or equal to 3. Most of the nuclei with fractal dimensions approaching 3 are stable, which are more like liquid drops.

In nuclear system, considering the self-similarity properties of the nuclear fractal system, we put forward a nuclear potential energy formula

$$u(r)=\frac{v_0D_\mathrm{f}}{3(D_\mathrm{f}-2)}(\frac{r}{r_\mathrm{s}})^{D_\mathrm{f}-3}\quad ,2<D_\mathrm{f}\leq 3. \tag{1}$$

which is proportional to the nuclear average density $\rho(r)$. r_s stands for the minimum scale size of a nuclear fractal system and it is also the maximum size of the minimum cluster element. v_0 is a coefficient and keeps constant, which is also corresponding to the estimation of the depth of nuclear potential well in liquid drop model when $D_\mathrm{f}=3$.

Next, we modify the phenomenological semi-empirical Bethe-Weizsäcker binding energy formula with the fermi gas model and the fractal theory. Here we mainly concern the liquid drop energy and put aside the correction term basing on the microscopic method, such as the description in[2]. The original one derived from liquid drop model is

$$B=(u_\mathrm{depth}-c_\mathrm{v}-c_\mathrm{as}(1-\frac{2Z}{A})^2)A-c_\mathrm{surf}A^{\frac{2}{3}}-c_\mathrm{Q}\frac{Z(Z-1)}{A^{\frac{1}{3}}}+c_\mathrm{p}\frac{(-1)^Z+(-1)^{A-Z}}{2A^{\frac{4}{3}}}, \tag{2}$$

where $u_\mathrm{depth}\approx 58$[3](the estimation of the depth of nuclear potential well); $c_\mathrm{v}=42.27$; $c_\mathrm{as}=23.48$; $c_\mathrm{surf}=17.72$; $c_\mathrm{Q}=0.72$; $c_\mathrm{p}=19.39$. We use the experimental mass data [4] to fit the other parameters.

The modified formulae are:

$$B_\mathrm{strong}=(v_\mathrm{depth}-c_1(\rho)-c_2(\rho)(1-\frac{2Z}{A})^2)A, \tag{3}$$

$$B_\mathrm{surf}=-c_s4\pi R^2, \tag{4}$$

$$B_\mathrm{Q}=-\frac{3}{5}\frac{Z(Z-1)e^2}{R}, \tag{5}$$

$$B_\mathrm{p}=c_\mathrm{p}\frac{(-1)^Z+(-1)^{A-Z}}{2A^{\frac{4}{3}}}, \tag{6}$$

$$B=B_\mathrm{strong}+B_\mathrm{surf}+B_\mathrm{Q}+B_\mathrm{p}, \tag{7}$$

where, $v_\mathrm{depth}=\frac{1}{2}(1-(\frac{A-(1+s)Z}{A})^2)\times\sum_{i=1}^{F}\frac{A_i}{A}(58+3u_i(R_i)\frac{D_\mathrm{f}-2}{D_\mathrm{f}})$; $c_1(\rho)=\frac{3}{5}\varepsilon(\rho)\rho^{\frac{2}{3}}$; $c_2(\rho)=\frac{1}{3}\varepsilon(\rho)\rho^{\frac{2}{3}}$; $c_\mathrm{s}=0.98\ \mathrm{MeV}\cdot\mathrm{fm}^2$; $c_\mathrm{p}=19.39$ MeV; $\varepsilon(\rho)=\varepsilon_0(\frac{\rho}{\rho_0})^\alpha$; $\varepsilon_0=264.12\ \mathrm{MeV}^{-1}$; $\rho_0=0.138\ \mathrm{fm}^{-3}$.

Besides, we define the total potential energy $U=U(A,Z,D_\mathrm{f},\rho)$, which is sum of the total nuclear potential energy $U_A=U_A(A,D_\mathrm{f},\rho)$ and the total electromagnetic potential energy $U_Z=U_Z(Z,D_\mathrm{f},\rho)$. Namely, $U=U_A+U_Z$.

The interaction among clusters in given nucleus is showed in Fig. 1. The total nuclear potential energy is

$$U_A = \sum_{i=1}^{F} (\frac{A_i}{A})^{\frac{3}{D_f}} U_i + \sum_{i=1}^{F} \sum_{i \neq j}^{F} (\frac{A_j}{A})^{\frac{3}{D_f}} U_{ij}, \tag{8}$$

where, $U_i = 2\pi \int_0^{\pi} \int_0^{R_i} u_i(r_i)\rho_{\rm dis}(r_i) r_i^2 {\rm d}r_i {\rm d}\theta$; $U_{ij} = 2\pi \int_0^{\pi} \int_0^{R_j} u_i(r)\rho_{\rm dis}(r_j) r_j^2 \sin(\theta){\rm d}r_{\rm j}{\rm d}\theta$; the distance $r = \sqrt{r_j^2 + R_{ij}^2 - 2r_j R_{ij}\cos\theta}$; $R_{ij} \approx R_i + R_j$. The total electromagnetic potential energy is

$$U_Z = \sum_{i=1}^{F} U_{Z_i} + \sum_{i=1}^{F} \sum_{i \neq j}^{F} U_{Z_{ij}}, \tag{9}$$

where, $U_{Z_i} = \frac{3}{5} Z_i(Z_i - 1)\frac{e^2}{R_i}$; $U_{Z_{ij}} = \frac{Z_i Z_j e^2}{R_{ij}}$.

For a given nucleus with NFCs in it, we consider that its fractal dimension $D_{\rm f}$ is constant. However, it should be a dynamical parameter in the process of nuclear synthesis. In a given nuclear reaction the binding energy $B = B(A,Z,D_{\rm f},\rho)$ and the total potential energy $U = U(A,Z,D_{\rm f},\rho)$ are changing with $D_{\rm f}$. When $D_{\rm f}$ gets the fixing value, then $B = B(A,Z,D_{\rm f},\rho)$ and $U = U(A,Z,D_{\rm f},\rho)$ get the minimum value, which corresponds to an interacting system becoming relatively stable. So one equation set is gotten:

$$\begin{cases} \partial_{D_{\rm f}} B(A,Z,D_{\rm f},\rho) = 0 \\ \partial_{D_{\rm f}} U(A,Z,D_{\rm f},\rho) = 0. \end{cases} \tag{10}$$

Because of the uncertain parameter α, we need more than two equations to study the structure properties in nucleus. The additional one is $B(A,Z,D_{\rm f},\rho) = B_{\rm exp} + E_{\rm excited}$, where $B_{\rm exp}$ is the experimental value of binding energy. And the $E_{\rm excited}$ is the change of the binding energy due to the nucleus being excited from the ground state, which is corresponding to the situation that the NFCs structure is forming in the excited nucleus. If only the nuclei in ground states are considered, $E_{\rm excited} = 0$.

Finally, we arrive at the modified binding energy formula $B(A,Z,D_{\rm f},\rho)$ and the total potential energy $U(A,Z,D_{\rm f},\rho)$, which are the functions of A, Z, $D_{\rm f}$ and ρ. And the important equation set (10) is gotten. Use(10), the additional equation and assume rational fractal structure types in nucleus, the nuclear fractal dimension and radius can be determined.

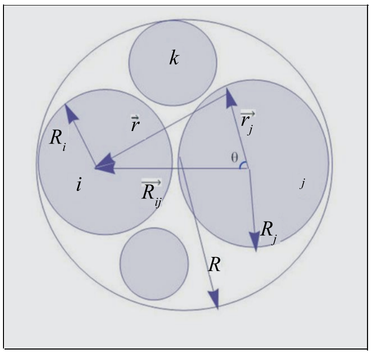

Fig. 1 (color online) The interaction among clusters in a given nucleus. The notations i,j,k stand for different NFCs.

In summary, we have done some calculations and obtained some results for light nuclei in ground states. It shows that the NFC structures are determined by the interactions within nuclear systems and that the fractal dimension can generally describe such structural features. The nuclear fractal mean density radii represent approximately the nuclear size, which are associated with the scale variables. Actually, the relations among the nuclear structure geometric variables are complex and correlative. We consider the importance that the nuclear irregular structure properties and the self-similarity characteristic may be the intrinsic aspects of nuclear structure properties. For the description of nuclear geometric properties, nuclear fractal dimension is an irreplaceable variable similar to the nuclear radius. Compared with the liquid drop model, it is a feature that the fractal description can reflect the important characteristics of the NFC structures especially for describing the nuclei far from the line of stability and α-cluster nuclei.

References

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