mentioning that the dimensional drop in the critical case exemplifies the degeneration studied in in a general setting.

References

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1 - 20 Kohn Condition and Exotic Newton Hooke Symmetry in Non-Commutative Landau Problem

Zhang Pengming and P. A. Horvathy

Kohn's theorem says that a system of charged particles in a uniform magnetic field can be decomposed into center-of-mass and relative coordinates if the charge-to-mass ratios are the same for all particles. For an isolated system, the possibility of having a center-ofmass decomposition relies on the non-trivial cohomology of the Galilei group. In $d\geqslant 3$ space dimensions the latter is the same as the one which generates the central extension by the mass. In the plane the Galilei group also admits an "exotic" central extension, though, highlighted by the non-commutation of boosts. "Exotic" Galilean symmetry is realized by non-commutative particles in the plane; such a particle can be coupled to an electromagnetic field, leading to the non-commutative Landau problem.

In this work we combine and extend these results to a system of N exotic particles in the plane. First we briefly review some aspects of the Landau problem for N ordinary particles. Our new results show that, in both the regular and singular cases, the motion is fully determined by the respective conserved quantities.

The intuitive meaning of the Kohn condition is to guarantee a collective behavior: all particles rotate with the same frequency, shared also by their center-of-mass. The additional condition $e_a\theta_a=$ const implies that the typical factors $(1-e_a\theta_a B)$ are the same for all particles, namely $(1-e\Theta B)$, allowing us to extend Kohn's theorem to exotic particles.

Our new result is to prove the two-parameter centrallyextended "exotic" Newton-Hooke symmetry for our system of N exotic particles. As in the Galilean case, the commutation relations only differ from the ordinary (1-parameter) case in the boost-boost relation, which now also involves the non-commutative parameter Θ, and is supplemented by internal rotations and time translations.

It is worth saying that all our investigations have been purely classical. It is not difficult to quantize our system, though, as in the ordinary case[1].

Reference

[1] G. W. Gibbons, C. N. Pope, Ann. Phys., 326(2011)1760.