1 - 21 Kohn's Theorem and Newton-Hooke Symmetry for Hill's Equations

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The relationship between the ability to split off the center-of-mass motion, the idea of a "guiding center" and its connection with some form of generalized Galilean, or Newton-Hooke-type, "kinematic symmetry[1]" has been the subject of a number of recent papers[2−3]. In the presence of magnetic fields this is the subject of Kohn's theorem and its variants. The use of the guiding center approximation in plasma physics is well known. Less well explored is the application of these ideas to gravitational physics. It is true that the idea of a guiding center is well established in galactic dynamics, but its connection with kinematic symmetries does not appear to have been explored before. The purpose of the present paper is to fill that gap in the literature.

The oldest example of what we have in mind are Hill's equations for the Earth-Moon-Sun system. However with the development of our understanding of the structure of the galaxy, it was realized that similar equations hold for the motion of stars around the Milky Way. Understanding many-electron atoms in the old quantum theory leads to the same equations and its failure to deal with the helium was the notorious stumbling block that led to the development of modern quantum mechanics. In more recent times there has been a revival of interest in semiclassical models of many-electron atoms and muonic atoms.

A remarkable aspect of Hill's equations is that our results simultaneously describe time-dependent symmetries and the motion of the center of mass. Our solutions represent therefore the trajectories both of the symmetry group acting on spacetime, and of the center of mass.

As long as we consider the 3-body problem it would be physically more important to study the relative-motion equation, the center of mass has little interest for, say, lunar motions. But Hill′s equations also arise when describing an electron beam in a synchrotron; guiding center motion is plainly interesting for the latter, as it is in plasma physics, or in stellar dynamics. Moreover, the motion of the center of mass can further be decomposed into that of the guiding center and relative motion; the generalization of the chiral decomposition is ideally suited for that. It is worth mentioning that our calculations could be extended to "exotic"(i. e. noncommutative) particles.

References

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