

# 1 - 22 Chiral Decomposition in Non-commutative Landau Problem

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In two remarkable papers Alvarez et al.<sup>[1]</sup> pointed out that two uncoupled 1d chiral oscillators yield, when combined, an interesting non-commutative system in the plane. The purely-magnetic “exotic” Landau problem (NCLP) can, in particular, be obtained for a suitable choice of the parameters. The inclusion of the electric field which, for non-commutative particles, also induces an anomalous velocity term (missed in Ref. [1]), is important, though, in the study of a Bloch electron, and for both the ordinary and the anomalous Hall effects, for example.

The quantum mechanical description can, just like in classical mechanics, be derived from the symmetry alone. This is analogous to the similar derivation of the H-atom spectrum from the group theory, using the  $O(4)$  dynamical symmetry. The transition in the critical case is analogous to the one described in “Chern-Simons mechanics”, obtained by turning off the kinetic term in the ordinary Landau problem.

In this paper we generalize the chiral oscillator-exotic Landau-problem correspondence to nonvanishing electric fields. The examples of a constant and a harmonic electric field are worked out explicitly. We also re-derive, along the lines indicated by Ref. [1], the recently found “exotic” Newton Hooke symmetry. This allows us to discuss the main properties of the NCL problem including its exotic Newton Hooke symmetry and its relation to the Hall effect. The “phase transition” when the magnetic field crosses a critical value determined by the noncommutative parameter is studied in detail.

Let us mention, in conclusion, that the chiral decomposition has recently been extended to the Hill equations of celestial mechanics<sup>[2]</sup>.

## References

- [1] P. D. Alvarez, J. Gomis, K. Kamimura, et al., Phys. Lett., B659(2008)906; Ann. Phys., 322(2007)1556.
- [2] P. M. Zhang, G. W. Gibbons, P. A. Horvathy, Phys. Rev., D85(2012)045031; P. M. Zhang, P. A. Horvathy, arXiv: 1202.5081 [hep-th].

# 1 - 23 On Gauge Invariant Nucleon Spin Decomposition

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It has been a long-standing problem of gauge invariant definition of gluon spin and orbital angular momentum. Recently a gauge invariant decomposition of the total nucleon angular momentum into quark and gluon constituents has been proposed<sup>[1]</sup>, and subsequently other possible gauge invariant decompositions for nucleon spin have been suggested<sup>[2]</sup>. Despite this progress there are still principal controversies on the fundamental conceptual level in determining a consistent notion for gluon spin and orbital angular momentum.

In this letter we revise the problem of nucleon spin decomposition and the existence of a consistent gauge invariant concept of spin in the non-Abelian gauge theory. We find that there is a wide number of possible gauge invariant spin decompositions suggested in<sup>[2]</sup>. In general they lead to gauge nonequivalent gluon spin operators. The Poincare and conformal invariance selects a unique Lorentz invariant decomposition with the Lorentz-type constraint for physical field. However, since this decomposition is not well defined on mass-shell, its physical meaning is unclear. For most of Lorentz non-invariant decompositions the definition of spin operator is frame dependent. We have shown that there is a class of gauge equivalent spin decompositions leading to gauge invariant gluon spin operators consistent with the helicity notion, so that such definitions of spin operators are frame independent. In general, the gauge invariant spin densities represent spatially and temporally non-local functionals. However, this non-locality has no physical meaning

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